Algebra

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1 Solve proportions

1.1 $\frac{v}{6} = \frac{36}{18}$

Step 1

Simplify the fraction by dividing top and bottom by 18

$$\frac{v}{6} = \frac{36/18}{18/18} = \frac{2}{1}$$

Step 2

Multiply each side by 6

$$6 \cdot \left(\frac{v}{6}\right) = 6 \cdot 2 \implies v = 12$$

1.2 $\frac{140}{735} = \frac{160}{x}$

Step 1

Simplify the left hand side by dividing by 35

$$\begin{aligned} \frac{140/35}{735/35} &= \frac{160}{x} \\ \frac{4}{21} &= \frac{160}{x} \end{aligned}$$

Step 2

Cross multiply

$$4 \cdot x = 160 \cdot 21 = 3360$$

Step 3

Divide by 4

$$\frac{4x}{4} = \frac{3360}{4} \implies x = 840$$

2 Factoring: Algebra 1

2.1 $6x^2 - 11x + 4$

Step 1

Multiply the leading coefficient and constant to find the factors that sum to 11. The leading coefficient is 6 and the constant is 4 so the product is 24. The factors of 24 are $((1, 24), (3, 8), (4, 6))$. Out of these, $3 + 8 = 11$.

Step 2

Break up the middle coefficient into the sum by writing $-11x = -8x - 3x$ so the equation becomes

$$6x^2 - 8x - 3x + 4$$

Step 3: Factor by grouping

$$\begin{aligned} 6x^2 - 8x - 3x + 4 &= 2x \cdot (3x - 4) - 1 \cdot (3x - 4) \\ &= (2x - 1)(3x - 4) \end{aligned}$$

2.2 $6x^2 - 19x - 7$

Step 1

Multiply the leading coefficient and constant and list the factors: $6 \cdot -7 = 42$. The factors of 42 are $((1, 42), (2, 21), (3, 14), (6, 7))$. Out of these, notice that $21 - 2 = 19$ which is the middle coefficient.

Step 2

Split the middle term up into the equation found in step 1:

$$6x^2 - 19x - 7 = 6x^2 - 21x + 2x - 7$$

Step 3: Factor by grouping

$$\begin{aligned} 6x^2 - 21x + 2x - 7 &= 3x \cdot (2x - 7) + 1 \cdot (2x - 7) \\ &= (3x + 1)(2x - 7) \end{aligned}$$

3 Exponents

3.1 $\left(x^5\right)^3$

Step 1

Multiply the two exponents so $(x^5)^3 = x^{15}$

3.2 $\left(3c^4)^2\right)$

Step 1

Make sure to square everything in the parentheses so $(3c^4)^2 = 3^2 \cdot (c^4)^2 = 9c^8$

4 Factor by: completing the square

4.1 $x^2 - 8x + 5 = 0$

Step 1

Subtract the constant term from both sides

$$x^2 - 8x + 5 = 0 \iff x^2 - 8x = -5$$

Step 2

Divide the coefficient of x by 2 and then square it. Add the result to both sides. The coefficient of x is 8, and $\frac{8}{2} = 4$. Adding 4^2 to both sides, the equation becomes

$$x^2 - 8x = -5 \iff x^2 - 8x + 16 = 11$$

Step 3

Factor the left hand side

$$x^2 - 8x + 16 = (x-4)^2$$

Step 4

Take the square root of each side:

$$(x-4)^2 = 11 \iff x - 4 = \pm\sqrt{11} \iff x = 4 \pm \sqrt{11}$$

5 FOIL

When to apply FOIL: When you have two binomials! Or when you have something that looks like $(ax + b) \cdot (cx + d)$

5.1 $(3x - 7)(5x + 6)$

Step 1

First: $3x \cdot 5x = 15x^2$
Outside: $3x \cdot 6 = 18x$
Inside: $-7 \cdot 5x = -35x$
Last: $-7 \cdot 6 = -42$
Adding all of these together and combining like terms (Outside and Inside), we get the answer

$$(3x - 7)(5x + 6) = 15x^2 - 17x - 42$$

5.2 $(4x^2 - 9)(8x^2 + 3)$

Step 1

First: $4x^x \cdot 8x^2 = 32x^4$
Outside: $4x^2 \cdot 3 = 12x^2$
Inside: $-9 \cdot 8x^2 = -72x^2$
Last: $-9 \cdot 3 = -27$
Adding all of these together and combining like terms (Outside and Inside), we get the answer

$$(4x^2 - 9)(8x^2 + 3) = 32x^4 - 60x^2 - 27$$

5.3 $(3x^2 + 2y)(7x^2 + 3y)$

Step 1

First: $3x^2 \cdot 7x^2 = 21x^4$
Outside: $3x^2 \cdot 3y = 9x^2y$
Inside: $2y \cdot 7x^2 = 14x^2y$
Last: $2y \cdot 3y = 6y^2$
Adding all of these together and combining like terms (Outside and Inside), we get the answer

$$(3x^2 + 2y)(7x^2 + 3y) = 21x^4 + 23x^2y + 6y^2$$

6 Factor with: Greatest Common Factor

6.1 $3x + 15$

The GCF is 3 so it factors as $3(x + 5)$

6.2 $16x^2 - 12x$

The GCF is $4x$ so it factors as $4x(4x - 3)$

6.3 $15x^3y^2 + 10x^2y^4$

The GCF is $5x^2y^2$ so it factors as $5x^2y^2(3x + 2y^2)$

6.4 $12x^5 - 18x^3 - 3x^2$

The GCF is $3x^2$ so it factors as $3x^2(4x^3 - 6x - 1)$

7 Factor trinomials

7.1 $x^2 + 5x + 6$

Step 1

Find 2 number that multiply to the constant term and add up to the coefficient of the linear term, x. In this case, the factors of 6 are $(2, 3), (1, 6), (-2, -3), (-1, -6)$. The choice that adds up to 5 is $(2, 3)$

Step 2

Use these two numbers fo factor the trinomial as

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

7.2 $x^2 + 4x - 32$

Step 1

The coefficent -32 factors as $(1, -32), (-1, 32), (2, -16), (-2, 16), (4, -8), (-4, 8), (6, -6)$. Notice that $(-4 + 8) = 4$ so these are teh two numbers we are looking for.

Step 2

Applying step two we can use these two number to factor the trinomial as

$$x^2 + 4x - 32 = (x - 4)(x + 8)$$

7.3 $x^2 + 7x + 10$

Step 1

The factors of the coefficient 10 are $(1, 10), (-1, -10), (2, 5), (-2, -5)$. Notice that $2 + 5 = 7$ so these are the two numbers we are looking for.

Step 2

We can use these two numbers to factor the trinomial as

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

8 Factor using: grouping

8.1 $xy + 2x + 3x + 6$

Step 1

Separate the terms into groups that share common factors:

$$xy + 2y = y(x + 2)$$
$$3x + 6 = 3(x + 2)$$

Step 2

Factor out the common factor $(x + 2)$ to get

$$xy + 2x + 3x + 6 = (y + 3)(x + 2)$$

8.2 $4x^2 + 2x^2 - 2x - 1$

Step 1

Separate the terms into groups that share common factors:

$$4x^2 + 2x^2 = 2x^2(2x + 1)$$
$$-2x - 1 = -1(2x + 1)$$

Step 2

Factor out the common factor $(2x + 1)$ to get

$$4x^2 + 2x^2 - 2x - 1 = (2x^2 - 1)(2x + 1)$$

8.3 $2x^2 + ay - ax^2 - 2y$

Step 1

Separate the terms into groups that share common factors:

$$2x^2 - 2y = 2(x^2 - y)$$
$$ay - ax^2 = -a(x^2 - y)$$

Step 2

Factor out the common factor $(x + 2)$ to get

$$2x^2 + ay - ax^2 - 2y = (2 - a)(x^2 - y)$$

9 Simplify each expression

9.1 $3(x+3)-2x$

Step 1: distribute

$3(x+3)-2x = 3x+9-2x$

Step 2: combine like terms

$3x+9-2x = x+9$

9.2 $2b-4(2b+5)$

Step 1: distribute

$2b-4(2b+5) = 2b-8b-20$

Step 2: combine like terms

$2b-8b-20 = -6b-20$

9.3 $t-4(t-8)+7(3t+4)$

Step 1: distribute

$t-4(t-8)+7(3t+4) = t-4t+32+21t+28$

Step 2: combine like terms

$t-4t+32+21t+28 = 18t+60$

10 Solve for x: linear equations

10.1 3x + 5x + 4 - x + 7 = 88

Step 1: turn into form $ax + b = c$

Combine like terms on each side to turn the equation into the form $7x + 11 = 88$

Step 2: Isolate x by subtracting

$$7x + 11 = 88 \iff 7x = 88 - 11 = 77$$

Step 3: Divide to isolate x completely

$$7x = 77 \iff x = 11$$

10.2 $\frac{x}{4} + \frac{x}{2} + 12 = 24$

Step 1: turn into form $ax + b = c$

Combine like terms to get $\frac{3x}{4} + 12 = 24$

Step 2: Isolate x by subtracting

Continue to isolate x by subtracting

$$\frac{3x}{4} = 12$$

Step 3: Multiply to isolate x completely

Multiply by the reciprocal $\frac{4}{3}$ to isolate x completely

$$\left(\frac{4}{3}\right)\left(\frac{3}{4}x\right) = \left(\frac{4}{3}\right)12 \iff x = 16$$

11 Quadratic formula

The quadratic formula gives the solution to a quadratic equation of the form $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

11.1 $x^2 + 6x = 14$

Step 1: put in appropriate form

$x^2 + 6x - 14 = 0$

Step 2: Plug into the equation

Here $a = 1, b = 6, c = -14$ so

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-14)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 56}}{2} = \frac{-6 \pm \sqrt{92}}{2}$$

Step 3: simplify

$$x = \frac{-6 + \sqrt{92}}{2} = -3 + \frac{2}{2}\sqrt{23} = -3 \pm \sqrt{23}$$

11.2 $4x^2 - 6x = 14$

Step 1: put in appropriate form

$4x^2 - 6x - 14 = 0$

Step 2: Plug into the equation

Here $a = 4, b = -6, c = -14$ so

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(-14)}}{2(4)} = \frac{6 \pm \sqrt{36 + 224}}{8} = \frac{66 \pm \sqrt{260}}{8}$$

Step 3: simplify

$$x = \frac{6 + \sqrt{260}}{8} = \frac{3}{4} + \frac{2}{8}\sqrt{65} = \frac{3 \pm \sqrt{65}}{4}$$

12 Solving inequalities

Similar to solving equations

12.1 $x + 2 > 12$

Step 1

Subtract 2 from each side

$$x + 2 > 12 \iff x > 10$$

12.2 $3x < 10$

Step 1

Divide each side by 3 (when dividing by a positive number the direction of the inequality stays the same)

$$x < \frac{10}{3}$$

12.3 $5 - 4x > 9$

Step 1

Subtract 5 from each side to get $-4x > 4$

Step 2

Divide each side by -4 (IMPORTANT: when dividing by a negative number, the direction of the inequality switches)

$$\frac{-4x}{4} > \frac{4}{-4} \iff x < -1$$

13 Solving systems of equations by substitution

13.1 $-4x + 3y = -2; y = x - 1$

Step 1

Set up the system of equations like

$$\begin{aligned} -4x + 3y &= -2 \\ y &= x - 1 \end{aligned}$$

And plug in y to the first equation so that there is only one variable, x. The first equation becomes

$$-4x + 3(x - 1) = -2$$

Step 2

Solve for x

$$\begin{aligned} -4x + 3x - 3 &= -2 \\ -x - 3 &= -2 \\ x &= 1 \end{aligned}$$

Step 3

Plug the x-value into the second equation to solve for Y $y = x - 1 = -1 - 1 = -2$

Step 4

The answer is $x = -1, y = -2$ or $(-1, -2)$

14 Solving systems of equations by elimination

14.1 $2x + 3y = 20; -2x + y = 4$

Step 1

Set up the system of equations

$$2x + 3y = 20$$
$$-2x + y = 4$$

In order to use elimination, you must rearrange the equations to create variables that have the same coefficients (these equations already do)

Step 2

Add the equations:

$$(2x + 3y) + (-2x + y) = (20 + 4) \implies 4y = 24 \implies y = 6$$

step 3

Plug in the y-value to find x

$$-2x + y = 4 \implies -2x + 6 = 4 \implies -2x = -2 \implies x = 1$$

Step 4

The solution is $x = 1, y = 6$ or $(1, 6)$

14.2 $-5y + 4x = 49; 7y + 2x = -23$

Step 1

Set up the system of equations

$$-5y + 4x = 49$$
$$7y + 2x = -23$$

Created variables with the same coefficents. The coefficient of x in the first equation is 4 and in the second equation is 2, so we can multiply the second equation by 2 to created the same coefficients. Doing this, the second equation becomes

$$14y + 4x = -46$$

Step 2

We can now subtract the second equation to eliminate x and the equation becomes

$$\begin{aligned}(-5y + 4x) - (14y + 4x) &= 49 - (-46)\\ -19y &= 95\\ y &= -5\end{aligned}$$

Step 3

Plug in the y value into the original equation and find x

$$\begin{aligned}7y + 2x &= -23\\ 7(-5) + 2x &= -23\\ 2x &= 12\\ x &= 6\end{aligned}$$

so the solution is $x = 6, y = -5$ or $(6, -5)$

15 Distance formula

For points (x_1, y_1) and (x_2, y_2) the distance between them is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

15.1 Find the distance between (3,5) and (8,2)

Step 1: Plug into the formula

Here $x_1 = 3, y_1 = 5, x_2 = 8, y_2 = 2$ and

$$d = \sqrt{(8-3)^2 + (2-5)^2} = \sqrt{25+9} = \sqrt{36} = 5$$

15.2 Find the distance between (25,39) and (19,43)

Step 1: Plug into the formula

Here $x_1 = 25, y_1 = 39, x_2 = 19, y_2 = 43$ and

$$d = \sqrt{(25-19)^2 + (34-43)^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$$

16 Simplify the following expression

16.1 $|3| + |-3| = ?$

$3 + 3 = 6$

16.2 $|x| + 3 = ?, x = -9$

$|-9| + 3 = 9 + 3 = 12$

16.3 Find x for $|3x + 9| = 6$

Since there is an absolute value we need to solve two equations

$$3x + 9 = 6$$
$$3x + 9 = -6$$

Solvin the first one, we see that

$$3x = -3 \implies x = -1$$

Solving the second one, we see that

$$3x = -15 \implies x = -5$$

so the solutions are $x = -1, -5$

16.4 Find x for $-2|3x + 9| = 6$

Divide both sides by -2 so get the equation $|3x + 9| = -3$. Absolute values are never negative so there is **NO SOLUTION**

17 Multiply and divide monomials

17.1 $9c^2 \times 5c$

Step 1: Multiply by multiplying the constants and the variables

$9 \cdot 5 \cdot c^2 \cdot c$

When multiplying variables, add the exponents together to get the product.

$c^2 \cdot c^1 = c^{2+1} = c^3$

Step 2

The solution is $45c^3$

17.2 $\frac{4q^3}{q^2}$

Step 1: Divide $\frac{q^3}{q^2}$

When dividing variables, subtract the top exponent by the bottom exponent.

$\frac{q^3}{q^2} = q^{3-2} = q^1 = q$

Step 2: Don't divide 4 by anything because there is not another constant

The solution is $4q$

18 Negative exponents

18.1 x^{-2}

Step 1: Take the reciprocal

A negative exponent means you need to flip the base to the other side.

$\frac{x^{-2}}{1} = \frac{1}{x^2}$

18.2 $\frac{1}{-17^{-6}}$

Step 1: Reciprocal

-17^6